Dense bipartite patches in triangle-free graphs

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In the very early days of graph theory, one could be forgiven for believing that graphs that do not contain a three pairwise adjacent vertices are bipartite, or perhaps close to bipartite (like having bounded chromatic number). Of course, by now, we know better! Indeed, we have a rather well-developed understanding of the structure of triangle-free graphs. However, surprisingly, the following natural intuitive question has, till now, not been pursued: could there be some "dense enough" bipartite patches in triangle-free graphs?

More specifically, we have recently formulated the following conjecture along these lines:

Conjecture 1 *There exists $C > 0$ such that, for any triangle-free graph of minimum degree d, there exists a bipartite induced subgraph of minimum degree $C \log d$.*

If true, this would be sharp by a probabilistic construction. The best lower bound we know so far is 2.

For further information and other results related to this conjecture, please consult `https://arxiv.org/abs/1802.03727`.